PEAK LOAD REDUCTION ON ELECTRIC POWER DISTRIBUTION BY COORDINATION OF RESIDENTIAL LOAD MANAGEMENT

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Abstract

This work deals a new approach to coordinate the residential equipments management for optimising the load curve on electric power distribution systems in terms of peak load reduction. The load profile smoothing has the potential to improve the efficiency of operations and investments in the electric power distribution.

Among the other residential equipments, the electric heating can be modelled mathematically. A “bi-objective” optimisation method can mitigate between cost and comfort while respecting fixed constraints. The flexibility of this equipment can be exploited to smooth the load curve by moving their consumption from on-peak times to off-peak times.

This present approach propose a method to coordinate electric heating consumption that takes into account estimated load curve consisted by other domestic equipments and customer temperature comfort. A statistical evaluation will be made to quantify the profit in term of peak reduction when consumption of the other equipments is randomly generated.

Index Terms -- Electricity demand management, residential heating, load aggregation, peak load reduction

INTRODUCTION

Electricity network operators are very interested by consumers’ behaviour for better managing their system. On their side, the consumers want to preserve their comfort and their profit.

The demand of electricity has increased with the new developments in the world. This has put the pressure on the power utilities to meet the increasing demand of costumers. One electric power distribution, one simple way for meet this demand is to reinforce the power system capacity. However, this solution need not only expensive investments for electricity distributors but also increases the cost per for the costumers. An innovative way is to operate on costumer level.

Then, it is interesting to study the management strategies for smoothing electricity demand curve preserving the quality of service.

Several methods to reduce residential peak load have been developed [2]. R. Teive and S. Vilvert, in [4], show how the application of DSM, in particular the load management, can be a very efficient technique for energy conservation. In [5], with Jorge, H. Antunes, and CH Martins, a multiple objective decision support formulation is proposed for aiding the dispatcher to select control strategies, which are to be applied to load groups. J.W. Black has investigated the system-wide implications of regulatory policies to promote demand reponse as a substitute for investments in system capacity (generation, transmission, and distribution) in [6].

On the first section, a typical residential load curve is used to generate randomly a load curve constituted by all equipments without electric heating.

On the second section, the electric heating consumption is modelled and an optimisation method is proposed. This equipment has a particular characteristic when compared with other ones: it can be temporarily turn off, decreased or increased without serious inconveniences for the customer. Their energy consumption is function of the characteristics of each house (insulation, thermal transfer), outside temperature and the selected tariff.
A new approach to manage the residential load is proposed in the third section. In this approach, the load control located at the level of the house. It consists in shifting the electric heating consumption for avoiding their simultaneous consumption with other residential equipments. Finally, the effect of this strategy in terms of peak load reduction for one house and over a group of several houses is investigated.

I. GENERATION OF RESIDENTIAL LOAD CURVE

The consumers themselves do not know the chronology of the calls of power induced by their consumption. To study it, it will be necessary to have a "tight interrogation" of each member of the families but it is practically impossible. The load aggregation notion is very complicated because each household has never a same load curve.

To simplify this study, a typical residential profile is used for reference. It is needed to generate randomly the residential profile without electricity heat consumption. In [8, 9, 10, 11], the analysis of the series of measurement enabled us to have data on the residential load curve.

The following table represents, for each time period, a mean (µ) and a deviation (σ) of a typical residential load profile without electric heating.

<table>
<thead>
<tr>
<th>Period [h]</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>µ [W]</td>
<td>500</td>
<td>300</td>
<td>200</td>
<td>250</td>
<td>1000</td>
<td>1800</td>
<td>2800</td>
<td>4500</td>
<td>4000</td>
<td>2000</td>
<td>2500</td>
<td>3500</td>
</tr>
<tr>
<td>σ [W]</td>
<td>200</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>500</td>
<td>500</td>
<td>800</td>
<td>800</td>
<td>800</td>
<td>500</td>
<td>500</td>
<td>700</td>
</tr>
<tr>
<td>Period [h]</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>µ [W]</td>
<td>3000</td>
<td>1600</td>
<td>1700</td>
<td>3000</td>
<td>5000</td>
<td>7500</td>
<td>7000</td>
<td>6500</td>
<td>5500</td>
<td>3500</td>
<td>1500</td>
<td>500</td>
</tr>
<tr>
<td>σ [W]</td>
<td>700</td>
<td>300</td>
<td>500</td>
<td>700</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>700</td>
<td>500</td>
<td>300</td>
</tr>
</tbody>
</table>

Table1: Means and deviations of typical residential load profile

Using a normally random generation with Matlab statistics Toolbox, an example for 20 curves generated randomly are plotted on these following figures.

These figures show the diversity of power consumption for each house.

II. ELECTRIC HEATING: MODELISATION AND OPTIMIZATION

In France, the electric heating accounts for 10% of national electricity consumption and 36% of the household consumption. It equips 30% in residential sector. Thus, the coordinating management of the electric heating consumption makes possible to carry out important economies of energy.
This type of equipment was studied separately because its inertial thermal characteristic can be exploited for residential load control. Electric heating consumption is much related to outdoor temperature and thermal characteristic of the house. It can be temporarily disconnected or decreased without significant modification of the indoor temperature. The sum of the quadratic differences between ideal indoor temperature levels and actual level will be used for comfort indicator. This comfort must be respected to preserve the quality of service when the electric heating control strategy is applied.

The optimization problem of the electric space heating is based on two objectives [3].

II-1. Modelisation

Dynamic of house

Houses are considered as heat storage subject to losses due to ventilation, convection and radiation. In this model, the state of house in period \( t_i \) is the amount of heat energy \( Q_i \) in kWh stored in. A discrete time model will be used with a time interval \( \Delta t \) of one hour. The dynamics of the house are described by changes in the total heat energy.

\[
Q_i = Q_{i+1} + q_{i+1} \cdot \Delta t - \text{Losses}_{i+1}
\]

with:

\[
\text{Losses}_{i+1} = \alpha \cdot \Delta t \cdot (T_{i} - \text{Text}_i)
\]

It is the heat transfer out of the house

\( \alpha \) [kWh/°C]: is a constant describing the heat transfer

\( T_i \): indoor temperature during the period \( t_i \)

\( \text{Text}_i \): ideal indoor temperature during the period \( t_i \)

The temperature during the period \( t_i \) can be written with the following expression.

\[
T_i = \beta Q_i
\]

\( \beta \) [°C/kW]=1/C: is the inverse of thermal capacity

With the expressions (2) and (3), (1) can be written:

\[
T_i + (\alpha \cdot \beta \cdot \Delta t - 1)T_{i+1} - \beta \cdot \Delta t \cdot q_{i+1} = \alpha \cdot \beta \cdot \Delta t \cdot \text{Text}_{i+1}
\]

\( \Delta t \) [h]: is the time interval (one hour)

Optimisation problem

The first objective is to minimize the total heating costs during one day.

\[
\text{Min} \sum_{i} p_i \cdot q_i
\]

\( p_i \): electricity price for the period \( t_i \)

\( q_i \): heating consumption during the period \( t_i \)

\( N \): in our study, we take \( N=24 \) (hours)

The second objective is to minimize the total degradation of living comfort. It is represented by the following expression:

\[
\text{Min} \sum_{i} (T_i - \text{Text}_i)^2
\]

For normal days, it is appropriate to introduce a periodicity constraint for indoor temperature as:

\[
T_0 = T_N
\]

i.e. the indoor temperature at the last discretization point of the previous day is required to be equal to the indoor temperature at the last discretization point of the day.
By using a weighting method of two objectives, we can write the optimization problem of the electric space heating consumer as the following quadratic problem:

\[
\begin{align*}
\min_{i,T_i} & \sum_{i=0}^{N-1} p_i . q_i + \lambda_{i+1} \sum_{i=0}^{N-1} (T_{i+1} - T_{ref_{i+1}})^2 \\
\text{subject to:} & \\
T_0 &= T_N \\
T_i + (\alpha . \beta . \Delta t - 1) T_{i+1} - \beta . \Delta t . q_{i+1} &= \alpha . \beta . \Delta t . Text_{i+1} \\
T_{min} &\leq T \leq T_{max} \\
q_{min} &\leq q \leq q_{max}
\end{align*}
\]

\(\lambda_i\) is the consumer's willingness to trade cost savings to living comfort at time point \(t_i\).

II-2. Mathematical resolution:

This optimisation problem can be written in following quadratic problem form:

\[
\begin{align*}
\min_{i,T_i} & \frac{1}{2} T^T Q T + c^T T + p^T q \\
\text{subject to:} & \\
T_i &\leq T \leq T_{max} \\
q_{min} &\leq q \leq q_{max}
\end{align*}
\]

The equality constraints in the expression (4) can be written:

\[
A x = B
\]

with:

\[
A = \begin{bmatrix}
1 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 \\
0 & \alpha \beta \Delta t - 1 & 1 & \cdots & 1 & 0 & \cdots & \alpha \beta \Delta t - 1 \\
0 & \alpha \beta \Delta t - 1 & 1 & \cdots & 1 & 0 & \cdots & \alpha \beta \Delta t - 1 \\
0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0
\end{bmatrix}
\]

\(\lambda_i\): is the consumer's willingness to trade cost savings to living comfort at time point \(t_i\).
\[
B = \begin{pmatrix}
0 \\
\alpha \beta \Delta T_0 \\
\vdots \\
\alpha \beta \Delta T_{i-1}
\end{pmatrix}
\]

\[A \in \mathbb{R}^{2i+1 \times 2i+1} \text{ and } B \in \mathbb{R}^{i+1}\]

The “Quadratic programming” of Matlab is used to solve this problem.

II-3. Example

In this example, the outdoor temperature variation is presented in fig 2. There are also two levels of price of kWh during one day (fig 3). Ideal indoor temperatures are 16°C (from 10 pm to 7 am) and 20°C (from 8 am to 9 pm), i.e. a temperature profile desired by the consumers. The minimal and maximal temperatures are respectively 15°C and 18°C during the night, 18°C and 22°C during the day.

Data

The present house is equipped with the radiators with a maximum power \(q_{\text{max}} = 5\) kW, with following characteristics: \(\alpha \text{[kWh/°C]} = 0.09\), \(\beta \text{[°C/kWh]} = 0.6\), \(\Delta t = 1\).

Outside temperature and the electricity price variations

![Fig2: Example of outside temperature variation](image1)

![Fig3: Price of kWh during one day](image2)

Simulations

One result with \(\lambda = 0\) and one with \(\lambda = 0.03\) are presented in the following figure. The inside temperature and the corresponding electric heater consumption are plotted in Fig 4.
Fig4: Indoor temperatures variation and heating power consumption

The cost of electric heating in the first case is 2.73 euros. In the second, it is 3.07 euros. The cost difference between these two cases is approximately 34 cents of euros (for one day). The cost saving for one month is around 10 euros. Therefore, the coefficients $\lambda$ are significant to observe the consumer response.

II-4. Optimal solutions of compromise

In the following figure, the Pareto optimal curve is plotted by varying values of $\lambda$ between 0 and 1 with a step of 0.01. It presents Pareto points for criteria heating cost ($COST$) and degradation of living comfort ($INCONF$).

$$INCONF = \sum_{i=1}^{N} (T_i - T_{ref})^2$$

$$COST = \sum_{i=0}^{N-1} p_i \cdot q_i$$

Fig5: Optimal solutions between cost and comfort degradation

This figure enables us to choose the ideal solution (solutions of compromise) which answers the fixed objective.
III. STRATEGY OF RESIDENTIAL LOAD MANAGEMENT

The aim of the management action of the residential load is to smooth the load profile by shifting the consumption of certain equipment without damaging the comfort of the users. An action that consists in controlling the daily consumption of electric heating to limit the peak load by avoiding his simultaneous consumption with all other electrical appliances is proposed. This method takes into account that temperature remains within the band of fixed temperature although the heating is temporarily controlled (turned off or decreased) during one day. It consists to advance or to differ the hours from consumption and to vary the power levels to consume heating in order to limit total consumption with other equipments. However, for these other equipments, their global load profile is completely random during one day (cf section I). Then, daily generated profile of all other equipments is needed to calculate the optimal profile of electric heating consumption.

The following figure resumes the strategy of coordination load management proposed in this paper.

![Diagram of residential load management strategy]

The following stages summarize the steps of the main approach.

**Stage 1:**
- Generate a typical profile of one house without the electric heating.
- Model the dynamic of the house and calculate the profile of electric heating consumption (optimized by taking account of the price variation and the previous outdoor temperature during one day).
- These two profiles form total curve of one house with electric heating **without load control**.

**Stage 2:**
- Optimize the daily consumption profile of the heating: optimization with **new constraints** (power of threshold and load curve generated above)

**New constraints:**
The last constraints in (7) are modified by taking into account the daily estimated profile calculated.
The heating power consumption is limited by new constraints. Its can be written by following expression and used in optimisation problem (4).

\[ q_{\text{max}}(i) = P_{\text{max}}(i) - Peqts(i) \quad (i = 1, ..., 24) \]  

(8)

Where \( P_{\text{max}}(i) \) is the power of threshold and \( Peqts(i) \) is the power consumption (without electric heating) during the period \( t_i \).

- With the typical profile of one house (Stage 1) and this new profile of the heating, we have a total curve with load control.

**Stage 3:**
Probabilistic approach to analyze the effect in term of peak reduction by making several simulations with \( N \) numbers of houses: plot the histograms of total consumption for each time periods.

**III-1. IMPACT EVALUATION OF THE ACTION OF LOAD MANAGEMENT FOR \( N \) HOUSES**

All strategies of demand side management run up against the problem of profits evaluation that can induce. The historical demand, resulting from several simulations, can be approximated by a continuous distribution of probability. The selected mathematical distribution form is function of the data.

A normal distribution is specified by two parameters: the average \( \mu \) and the variance \( \sigma^2 \). These two parameters of the distribution of the population can be estimated on a sample.

The average can be written by:

\[ \mu = \frac{1}{N_{\text{Sum}}} \sum_{i=1}^{N_{\text{Sum}}} P_i \]  

(9)

The variance can be written by:

\[ \sigma^2 = \frac{1}{N_{\text{Sum}} - 1} \sum_{i=1}^{N_{\text{Sum}}} (P_i - \mu)^2 \]  

(10)

The function of density of the normal distribution is given by:

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right] \]  

(11)
III-2. SIMULATION AND RESULTS

Different stages of simulations are presented on this following chart:

<table>
<thead>
<tr>
<th>Data acquisition:</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Characteristic of the house</td>
</tr>
<tr>
<td>- Ideal indoor temperature</td>
</tr>
<tr>
<td>- Different constraints (Tmin, Tmax, qmax,...)</td>
</tr>
</tbody>
</table>

- Generate load curve for one house \( Peqts(i) \)
- Fix the maximum power of threshold \( P_{max}(i) \)
- Calculation of news constraints for the heating
  \[
  q_{max}(i) = P_{max}(i) - Peqts(i) \quad (i = 1, \ldots, 24)
  \]
- Optimal heating calculation for one house
  Optimization of electric heating consumption by taking account of the other residential equipments with news constraints
  \[
  q_{chauff}(i) \leq q_{max}(i) \quad (i = 1, \ldots, 24)
  \]
- Total load curve calculation without and with control
  *Without control*: generated load curve for one house + heating curve
  *With control*: generated load curve for one house + optimal heating curve

Fig8: Stages of simulation

Power electric heating was calculated to heat less during the night (15 with 18°C) and much more during the day (around 20°C) for reason of comfort. Electric heating consumption is added here with an average load curve but high peaks are observed. The following figures (*Fig9* and *Fig10*) show two results of two examples of hypothetically identical houses (*house1* and *house2*).

**Without control**
The total load curve (heating and load curve of all other equipments) and the corresponding variation of the temperature during one day (minimum cost with constraint of temperature) are plotted (on *Fig9* and *Fig10*).

**With control**
Considering the new constraints i.e. power of threshold and load curve of the equipments except heating, the daily profile of the heating consumption is re-optimised. The new optimal profile of total load curve (new optimal heating and load curve of all other equipments) and the corresponding variation of the temperature during one day are plotted (on *Fig9* and *Fig10*).
House 1:

![Load curve without and with control (house 1)](image1)

Fig 9: Load curve without and with control (house 1)

House 2:

![Load curve without and with control (house 2)](image2)

Fig 10: Load curve without and with control (house 2)

With control, the reduction of peaks is observed on two cases (house 1 and house 2). The total load curves are smoothed but the corresponding daily variation of inside temperature is different compared to the initial variation. This difference is because of heating consumption modification observed on Fig 9 and Fig 10 (load shifting). A high consumption of electric heating is observed before the peak periods to be able to reduce them. The temperatures variations respect the upper and the lower fixed temperatures constraints.

The following figure represents an illustration of the improvement load curve when this present strategy of residential load management is applied on 20 load curves generated randomly.
Impact of load control for a group of N houses

In this example, 10 houses have hypothetically the same characteristic. The aggregate load curve of 10 houses is obtained by making the sum of 10 curves obtained randomly as in the one house case.

By holding account this characteristic random with each generation, a high number of generations must be proceeded to obtain a more representative curve.

1000 curves are generated with this method and averages of aggregates global load curves for the two cases are plotted on figure 12: without control (in solid line) and with control (in dashed line).

The following figures represent the histograms of global powers consumption for 8h, 9h, 18h, 19h and 20h. They represent statistically the morning and the evening peaks reductions.
**Morning peak reduction**

![Fig13: Histograms of power consumption at 8h and 9h](image)

**Evening peak reduction**

![Fig14: Histograms of power consumption at 18h, 19h and 20h](image)

These figures show that the distribution of heating consumption each time point after 1000 simulations follows a normal law which the averages and the standard deviations can be calculated.
From 8h to 9h and 18h to 21h, the means and the standard deviations (see section III-1) of the aggregate loads for 10 houses with and without control, after 1000 simulations, are registered in the following table:

<table>
<thead>
<tr>
<th></th>
<th>8h</th>
<th>9h</th>
<th>18h</th>
<th>19h</th>
<th>20h</th>
<th>21h</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Without control</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$ [kW]</td>
<td>81.89</td>
<td>60.65</td>
<td>91.18</td>
<td>87.04</td>
<td>82.76</td>
<td>69.28</td>
</tr>
<tr>
<td>$\sigma$ [kW]</td>
<td>2.60</td>
<td>2.63</td>
<td>3.27</td>
<td>3.09</td>
<td>3.02</td>
<td>2.20</td>
</tr>
<tr>
<td><strong>With control</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$ [kW]</td>
<td>61.18</td>
<td>57.71</td>
<td>78.01</td>
<td>73.48</td>
<td>69.47</td>
<td>63.57</td>
</tr>
<tr>
<td>$\sigma$ [kW]</td>
<td>1.16</td>
<td>1.85</td>
<td>3.13</td>
<td>2.76</td>
<td>2.33</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table2: Means and deviations power consumption at 8h, 9h, 18h, 19h, 20h and 21h

This table present around 20 % of peak reduction between 8h to 9h and around 13 % between 18h to 21h when the heating is controlled.

**VII. CONCLUSION**

This paper presents a new approach for reducing residential peak load using Direct Load Control of electric heating. Optimal consumption of electric heating is calculated by considering estimated daily load curve of other residential equipments. The final objective is to reduce peak load system during one day by avoiding the simultaneous consumption of the residential equipments.

The results of simulations carried out showed that residential peak load can be reduced if the customers could accept to modify indoor temperature i.e. their habits.

The savings by residential peak reduction is directly in favour of electricity distributors.

For future work, it would be interesting to find a way to compensate the consumers in order to incite them to take part in the load management programs.
VIII. BIBLIOGRAPHIE


